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An Application of Associated Single Decrement Model in Pension Theory

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Abstract

Actuarial valuation is necessary for the accruals of a company's retirement obligations to its employees. Several decrements such as withdrawal, death and disability are considered in the calculation. Accrued Liability and Normal Cost are projected on a yearly basis using deterministic and stochastic approaches. The deterministic process employs the traditional methods of life contingency mathematics while the stochastic method uses Monte Carlo simulations to derive projections. The goal of this paper is to present the stochastic method as an alternative to traditional methods, and show convergence of the results with that of the deterministic process as the number of simulations increases.

AMS Classification: 62P05, 91B30, 65C05
Key words: pension, Actuarial Mathematics, Monte Carlo Method, life contingencies

1 Introduction

In the Philippine setting, every Institution, both government and private, is mandated to provide retirement benefits to its regular employees under Republic Act No. 7641. The said law stipulates the minimum retirement benefit, but some of the companies are generous enough to give more than the requirement. It serves also as a form of appreciation in the part of the employer for their loyal employees who offered their service for years.

Philippine Accounting Standard No. 19 prescribes the accounting and disclosure by employers for employee benefits. The standard gives guidance on cost accruals to be recognized in the financial statements. It also specifically identifies the Projected Unit Credit method to be used in performing valuation [4]. Employers are not obligated to maintain a fund for their retirement liability but are encouraged to do so, through tax incentives. Actuarial valuation involves multiple decrements if the retirement plan includes death and disability benefits [1].

In the *Projected Unit Credit* (PUC), each period of service gives rise to an additional benefit entitlement and measures each unit separately to build up the final obligation. *Accrued Liability* (AL), also called Actuarial Liability, represents the portion of the cost of projected benefits allocated to past years of service. *Normal Cost* (NC), also called Current Service Cost, represents the portion of the cost of projected benefits allocated to the current year [6].

In this paper, traditional valuation methods of retirement benefits involving life contingencies are compared with Monte Carlo simulations of stochastic processes for convergence of results. Monte Carlo methods are more flexible computational tools when the underlying mortality assumptions undergo changes.

2 Methods

Our model data set consists of 50 employees with age ranging from 30 to 50. For simplicity, birth date, hire date and valuation date are assumed to be December 31. Thus, age and years of service are whole numbers. This set of employees is assumed to be a closed group, that is, there is no replacement for an employee who leaves the group due to decrements, in the case of stochastic method. We use the *PUC* [6] as our valuation method, with valuation date December 31, 2011. For the multiple decrement table, we use the Illustrative Service Table [3]. We also assume that the interest rate is 8%, the salary increase rate is 5%. Benefits are assumed to be paid at the end of each calendar year. The retirement benefit schemes are standard, shown in Table 1.

Table 1: Retirement benefit schemes for employees.

Benefit Type	Eligibility	Benefit
Normal Retirement	Age 60 with ≥ 5 years service	100% of monthly salary for every year of service
Early Retirement	Age 50 with ≥ 10 years service	100% of monthly salary for every year of service
Resignation	Years of Service	Percentage of Salary
	5 – 10	25%
	10 – 15	50%
	15 – 20	75%
	20 up	100%
Death		100% of monthly salary for every year of service
Disability		100% of monthly salary for every year of service

Let ω be the retirement age, v the discount factor, w the withdrawal decrement, d the death decrement, b the disability decrement, q_x^w the probability that a life of age x resigns or retires within the next year, q_x^d the probability that a life of age x dies within the next year, q_x^b the probability that a life of age x becomes disabled within the next year, and ${}_kp_x^{(\tau)}$ the probability that a life of age x survives against all decrements within the next k years, where the left subscript is dropped for $k = 1$. For a life of age x, the withdrawal cost is computed by [2]

$$\text{withdrawal cost } = R_x \times q_x^w \tag{1}$$

where R_x is the retirement or resignation benefit at age x. The death cost is given by

$$\text{death cost } = D_x \times (1 - q_x^w) \times q_x^d \tag{2}$$

where D_x is the death benefit. The disability cost is given by

$$\text{disability cost } = B_x \times (1 - q_x^w) \times (1 - q_x^d) \times q_x^b \tag{3}$$

where B_x is the disability benefit. An interpretation of this is that for an employee to avail the death benefit, the employee must be in active service when death happens and thus, withdrawal decrement was survived. In the same vein, a disability benefit is given to a living employee in active service who becomes disabled, hence the employee must overcome the withdrawal and death decrements.

The deterministic method uses standard life contingency methods. Starting from the group of employees as of valuation date of December 31, 2011, Accrued Liability and Normal Cost are computed [1]. Survival against all decrements is then factored in for each employee with their respective ages in calculating the Accrued Liability and Normal Cost for the next year, which is December 31, 2012 valuation. Probabilities of survival are based on the Illustrative Service Table [3] [7].

For an employee of age x, the probability that the employee stays in the group during the year is

$$p_x^{(\tau)} = (1 - q_x^w)(1 - q_x^d)(1 - q_x^b). \tag{4}$$

This factor is considered for the retirement costs of the next year, the year after, etc. until the employee reaches retirement age when the probability of withdrawal becomes 100%.

Monte Carlo simulation is performed for each employee for each year. The uniformly distributed random number generated is compared with probabilities from the Illustrative Service Table [5] [8]. As a result, an employee of age x may either resign or retire, die, become disabled, or stay with the group until the year after.

Retirement costs are calculated for each employee. Accrued Liability is computed as follows:

$$\begin{aligned} AL = N_\omega\, v^{\omega-x}\, {}_{\omega-x}p_x^{(\tau)} &+ \sum_{k=0}^{\omega-x-2} E_{k+x+1} v^{k+1}\, {}_kp_x^{(\tau)} q_{x+k}^w P_{AL} \\ &+ \sum_{k=0}^{\omega-x-2} R_{k+x+1} v^{k+1}\, {}_kp_x^{(\tau)} q_{x+k}^w\, P_{AL} \\ &+ \sum_{k=0}^{\omega-x-2} D_{k+x+1} v^{k+1}\, {}_kp_x^{(\tau)} q_{x+k}^d\, P_{AL} \\ &+ \sum_{k=0}^{\omega-x-2} B_{k+x+1} v^{k+1}\, {}_kp_x^{(\tau)} q_{x+k}^b\, P_{AL} \end{aligned}$$

where P_{AL} is the number of years of service rendered at valuation date or the total number of years of service at retirement date, N_ω is the normal retirement benefit and E_x the early

Table 2: Comparison of Accrued Liability obtained using deterministic methods and stochastic projections on the model employee data over a period of 10 years.

Year	Deterministic	Stochastic	% error
0	5,433,939.61	5,433,939.61	0.00
1	8,961,663.01	8,996,721.72	0.39
2	10,682,868.86	10,742,250.05	0.56
3	12,385,041.38	12,463,317.83	0.63
4	14,546,090.09	14,628,229.33	0.56
5	19,614,856.58	19,747,487.72	0.68
6	22,253,853.23	22,417,378.77	0.73
7	24,817,047.85	25,000,979.33	0.74
8	27,996,894.50	28,199,382.29	0.72
9	35,075,569.07	35,332,068.40	0.73
10	38,271,152.65	38,546,735.29	0.72

retirement benefit. Normal cost is computed as follows:

$$\begin{aligned} NC = N_\omega \, v^{\omega-x} \,{}_{\omega-x}p_x^{(\tau)} &+ \sum_{k=0}^{\omega-x-2} E_{k+x+1} v^{k+1} \,{}_kp_x^{(\tau)} q_{x+k}^{w} P_{NC} \\ &+ \sum_{k=0}^{\omega-x-2} R_{k+x+1} v^{k+1} \,{}_kp_x^{(\tau)} q_{x+k}^{w} \, P_{NC} \\ &+ \sum_{k=0}^{\omega-x-2} D_{k+x+1} v^{k+1} \,{}_kp_x^{(\tau)} q_{x+k}^{d} \, P_{NC} \\ &+ \sum_{k=0}^{\omega-x-2} B_{k+x+1} v^{k+1} \,{}_kp_x^{(\tau)} q_{x+k}^{b} \, P_{NC} \end{aligned}$$

where $P_{NC} = 1/T$, T being the total years of service at retirement [2].

3 Results and Discussion

Monte Carlo simulations were performed using the model data set, and averaged over 500, 2,000, and 75,000 runs. Table 2 shows comparisons of the Accrued Liability obtained using deterministic methods and stochastic (Monte Carlo) projections.

Figure 1 shows the convergence of Monte Carlo results for Accrued Liability when the number of simulations is increased from 500 to 2,000 to 75,000. For 2,000 or more simulations, the percent error was consistently below 1%.

The Normal Cost was calculated similarly, using a deterministic approach and a Monte Carlo approach that used up to 75,000 simulations. The results are presented in Table 3, with errors of less than 1%.

Figure 2 shows the convergence of Monte Carlo results for Normal Cost when the number of simulations is increased from 500 to 2,000 to 75,000. For 2,000 or more simulations, the percent error was consistently below 1%.

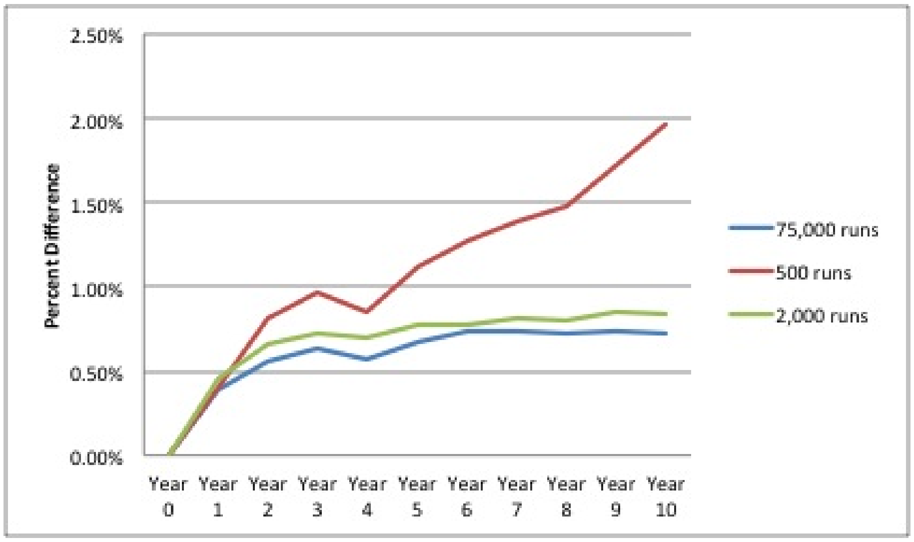

Figure 1: Monte Carlo simulations of Accrued Liability for 500, 2,000 and 75,000 runs

Table 3: Comparison of Normal Cost obtained using deterministic methods and stochastic projections on the model employee data over a period of 10 years.

Year	Deterministic	Stochastic	% error
0	1,472,726.22	1,472,726.22	0.00
1	1,630,306.53	1,637,196.96	0.42
2	1,711,275.76	1,721,343.75	0.59
3	1,794,681.93	1,806,448.88	0.66
4	1,892,995.03	1,904,734.55	0.62
5	2,065,220.81	2,079,926.23	0.71
6	2,173,306.39	2,189,779.08	0.76
7	2,277,941.95	2,295,141.77	0.76
8	2,397,219.32	2,415,494.43	0.76
9	2,598,699.49	2,618,376.17	0.76
10	2,683,647.33	2,703,413.94	0.74

4 Conclusion

In this paper we compared pension calculations using Monte Carlo simulations with traditional methods using life contingencies. The life contingencies method uses actuarial quantities in the the Illustrative Service Table derived from mortality tables, whereas Monte Carlo simulations simply depend on the mortality table. Monte Carlo methods are easier to implement, and flexible to changes in the mortality assumption. On the downside, Monte Carlo methods need a large number of simulations for better accuracy. Results showed that Monte Carlo simulations agreed with life contingencies methods within 1% when 75,000 simulations were used. Thus, Monte Carlo method is a reasonable alternative to traditional methods, particularly with availability of high speed computing.

For future studies, it would be recommended to consider an open group where there is replacement for an employee who leaves the cohort. Additional assumptions should be made

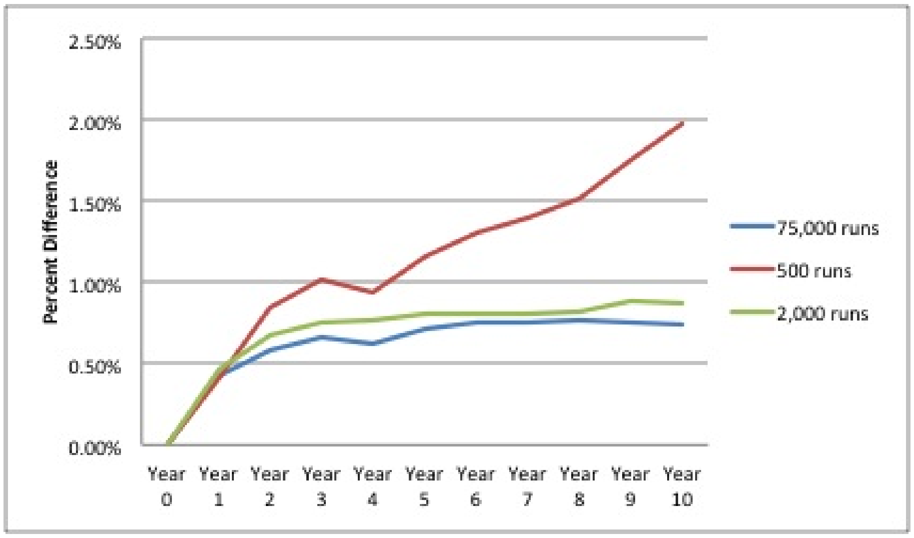

Figure 2: Monte Carlo simulations of Normal Cost for 500, 2,000 and 75,000 runs

as to the age and salary bracket of the new entrants. Fractional age assumption can also be included by not setting the age and years of service as whole numbers.

References

[1] Aitken, William H. A Problem-Solving Approach to Pension Funding and Valuation, 2nd Edition. Winsted, Connecticut: ACTEX Publications, 1996.

[2] Bowers, Newton L., Gerber, Hans U., Hickman, James C., Jones, Donald A., Nesbitt, Cecil J. Actuarial Mathematics, 2nd Edition. Schaumburg, IL: Society of Actuaries, 1997.

[3] Illustrative Service Table. Society of Actuaries, 2013. Retrieved from https://www.soa.org/Files/Edu/edu-2013-mlc-tables.pdf

[4] International Accounting Standards Committee. Employee Benefits. International Accounting Standard No. 19, 2009.

[5] Metropolis, Nicholas, Ulam, Stanislaw. "The Monte Carlo Method." J. Amer. Stat. Assoc. 44 (1949): 335–341.

[6] Smith, Matt. Projected Unit Credit. Office of the State Actuary, 2005.

[7] Weishaus, Abraham. Study Manual for Exam C/Exam 4, 10th Edition. Westbury, NY: Actuarial Study Materials, 2009.

[8] Wittwer, Jon W. Monte Carlo Simulation Basics, 2004. Retrieved from: http://vertex42.com/ExcelArticles/mc/MonteCarloSimulation.html