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Cycle and path extendability of $K_{1,3}-$free join of graphs

Jeric S. Alcala
Institute of Mathematical Sciences and Physics, College of Arts and Sciences
University of the Philippines
Los Baños, Laguna
jsalcala@up.edu.ph

Rolando E. Ramos
Institute of Mathematical Sciences and Physics, College of Arts and Sciences
University of the Philippines
Los Baños, Laguna

Abstract

A graph G is said to be cycle extendable if for every non-Hamilton-ian cycle C of G, there is a cycle C' in G such that $V(C) \subseteq V(C')$ and $|V(C')| = |V(C)| + 1$. If in addition, every vertex of G is in some triangle in G, then G is fully cycle extendable. On the other hand, we say that a graph G is path extendable if for every non-Hamiltonian $(u, v)-$path P of G, there is a $(u, v)-$path P' in G such that $V(P) \subseteq V(P')$ and $|V(P')| = |V(P)| + 1$. A graph is said to be $K_{1,3}-$free or claw-free if it does not contain a $K_{1,3}$ as induced subgraph. The join $G_1 + G_2$ of disjoint graphs G_1 and G_2 is the graph obtained from $G_1 \cup G_2$ by joining each vertex of G_1 with each of G_2. In this paper, we show that every $2-$connected $K_{1,3}-$free join of (two) graphs is cycle extendable and every $2-$connected $K_{1,3}-$free join of graphs is fully cycle extendable except $(K_1 \cup K_1) + (K_1 \cup K_m)$ for any integer $m \geq 1$. We also prove that every $3-$connected $K_{1,3}-$free join of graphs is path extendable except $(K_1 \cup K_n) + (K_r \cup K_s)$ for any positive integers n, r, s such that $n \geq 2$, $r \leq 2$ and $r + s \geq 3$.

Keywords: cycle extendable, path extendable, claw-free
Mathematics Subject Classification: 05C38; 05C45

1 Introduction and main results

Throughout this paper, we consider undirected, simple and finite graphs. For terminology and notation not defined here, we refer to [3]. We denote the vertex set and edge set of a graph G by $V(G)$ and $E(G)$, respectively. We used K_n, C_n and P_n as notations for complete graph, cycle, and path of n vertices, respectively. We used $G[S]$ to denote the subgraph of G induced by $S \subseteq V(G)$. A cycle or path containing all vertices of G is called a Hamiltonian cycle or Hamiltonian path, respectively. A path with end vertices u and v is called a $(u, v)-$path.

The union $G_1 \cup G_2$ of disjoint graphs G_1 and G_2 is the graph with vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$. The join $G_1 + G_2$ of disjoint graphs G_1 and G_2 is the graph obtained from $G_1 \cup G_2$ by joining each vertex of G_1 with each of G_2. If $G_2 \subseteq G_1$, the graph $G_1 \setminus G_2$ is the graph obtained by deleting every vertex of G_2 and every edges incident to those vertices from G_1. If $V(G_2) = \{v\}$, we write $G_1 \setminus G_2$ as simply $G_1 \setminus v$.

A graph G is said to be cycle extendable if for every non-Hamiltonian cycle C of G, there is a cycle C' in G such that $V(C) \subseteq V(C')$ and $|V(C')| = |V(C)| + 1$. In this case, we say C is extendable and C' is an extension of C in G. If in addition, every vertex of G is in some triangle in G, then G is fully cycle extendable. On the other hand, we say that a graph G is path extendable if for every non-Hamiltonian $(u, v)-$path P of G, there is a $(u, v)-$path P' in G such that $V(P) \subseteq V(P')$ and $|V(P')| = |V(P)| + 1$. Like in cycle extendability, we say P is extendable and P' is an extension of P in G.

Of particular interest in this paper are graphs which are claw-free. A complete bipartite graph $K_{1,3}$ is called a claw. A graph G is said to be claw-free if it does not contain a claw as induced subgraph. In this case, we call $K_{1,3}$ a forbidden subgraph of G. Many results on claw-free graphs, especially regrading their Hamiltonian properties, began to appear in 1980's. Most of the recent results are summarized in [4], [5]. Some results in path and cycle extendability involve the concept of connectivity and local connectivity. For instance, Wang and Zhu [9] showed that every connected, locally $2-$connected claw-free graph is path extendable. This result was generalized in [8] by relaxing the condition of locally $2-$connectedness using the notion of vertex cut. In [1], Arangno proved that every $2-$connected $K_{1,3}-$free chordal graph is cycle extendable. It was shown in [2] that every connected, locally connected $K_{1,3}-$free graph of order greater than 2 is fully cycle extendable. On the other hand, other results require an additional forbidden subgraph, other than the claw, to imply path and cycle extendability, as in [5] and in the following theorems:

Theorem 1.1. *[6] Every $2-$connected $K_{1,3}Z_2$-free graph is cycle extendable.*

Theorem 1.2. *[7] Every $2-$connected $K_{1,3}P_4$-free graph is fully cycle extendable except $(K_1 \cup K_1) + (K_1 \cup K_n)$ for any integer $n \geq 1$.*

Theorem 1.3. *[7] Every $3-$connected $K_{1,3}P_4$-free graph is path extendable except $(K_1 \cup K_m) + (K_1 \cup K_n)$ or $(K_1 \cup K_m) + (K_2 \cup K_n)$ for any integers $m, n \geq 1$.*

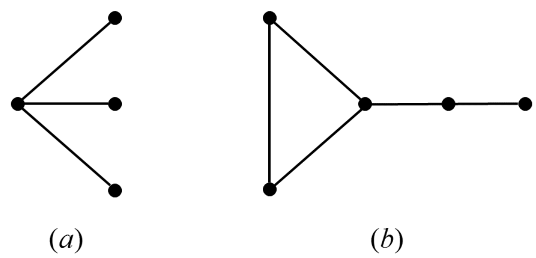

Figure 1: (a) claw or $K_{1,3}$, and (b) Z_2

In this paper, instead of considering additional forbidden subgraph, we limit the study to claw-free graphs which are join of two disjoint graphs. By considering this family of graphs, we obtained sufficient conditions for cycle and path extendability. These conditions are summarized in the following theorems, which are the main results of this paper.

Theorem 1.4. *Every* $2-$*connected* $K_{1,3}-$*free join of graphs is cycle extendable.*

Corollary 1.1. *Every* $2-$*connected* $K_{1,3}-$*free join of graphs is fully cycle extendable except* $(K_1 \cup K_1) + (K_1 \cup K_m)$ *for any integer* $m \geq 1$.

Theorem 1.5. *Every* $3-$*connected* $K_{1,3}-$*free join of graphs is path extendable except* $(K_1 \cup K_n) + (K_r \cup K_s)$ *for any positive integers* n, r, s *such that* $n \geq 2$, $r \leq 2$ *and* $r + s \geq 3$.

It is noticeable that we get the same exceptions for fully cycle extendability and path extendability as Ramos and Babierra [7]. Although not all $2-$connected (or $3-$connected) $K_{1,3}-$free join of graphs are $2-$connected (or $3-$connected) $K_{1,3}P_4-$free, and vice versa, by replacing the condition G being P_4-free by G being a join of two graphs we obtained the same results on fully cycle extendability and path extendability.

2 Proof of Theorem 1.4 and Corollary 1.1

Throughout this section, we let G be a $2-$connected $K_{1,3}-$free join of disjoint graphs G_1 and G_2. We suppose $C = c_0c_1..c_{k-1}c_0$ is a non-Hamiltonian cycle of G. We also use $\overrightarrow{c_ic_j}$ to denote the subpath of C from c_i to c_j, that is, $\overrightarrow{c_ic_j} = c_ic_{i+1}...c_j$ if $i < j \leq k-1$ or $\overrightarrow{c_ic_j} = c_ic_{i+1}...c_{k-1}c_0c_1...c_j$ if $j < i \leq k-1$. For convenience, subscripts for paths and cycles will be taken $\mod k$.

It is not difficult to see that if $k = 3$, then C is extendable. Also, it is easy to verify that for $k \geq 3$, if an edge of C is in G_1 and a vertex of G_2 is not in C then C is extendable because G is join of G_1 and G_2. It is left to show that C is extendable if either no edge of C is in either G_1 or G_2, or no edge of C is in G_1 but all vertices of G_1 is in C for $k > 3$. The proof for these two cases are presented in the following claims.

Claim 2.1. *If no edge of* C *is in* G_1 *or* G_2*, then* C *is extendable for* $k > 3$.

Proof: Since C is a non-Hamiltonian cycle, then there is a vertex z in either G_1 or G_2 not in C. Also, since $k > 3$, then we can find a subpath $c_ic_{i+1}c_{i+2}c_{i+3}$ of C such that $c_i, c_{i+2} \in V(G_u)$ and $c_{i+1}, c_{i+3} \in V(G_v)$ where $u \neq v$. WLOG, let $z, c_i, c_{i+2} \in V(G_1)$. Since G is $K_{1,3}-$free and $G = G_1 + G_2$, then at least one pair of vertices in $\{z, c_i, c_{i+2}\}$ must be adjacent. If $zc_i \in E(G_1)$ or $zc_{i+2} \in E(G_1)$, then $c_iz\overrightarrow{c_{i+1}c_i}$ or $c_{i+1}z\overrightarrow{c_{i+2}c_{i+1}}$ are extensions of C, respectively. If $c_ic_{i+2} \in E(G_1)$, then the cycle $c_{i+1}z\overrightarrow{c_{i+3}c_i}c_{i+2}c_{i+1}$ is an extension of C.

Claim 2.2. *If no edge of* C *is in* G_1 *and all vertices of* G_1 *are in* C*, then* C *is extendable for* $k > 3$.

Proof: Since C is a non-Hamiltonian cycle, then there exists a vertex y in G_2 not in C. Also, because no edge of C is in G_1, we can find a subpath $c_mc_{m+1}c_{m+2}$ of C such that $c_m, c_{m+2} \in V(G_2)$ and $c_{m+1} \in V(G_1)$. Because G is $K_{1,3}-$free then at least one pair of vertices in $\{c_m, c_{m+2}, y\}$ must be adjacent. If $yc_m \in E(G_2)$ or $yc_{m+2} \in E(G_2)$, then $c_my\overrightarrow{c_{m+1}c_m}$ or $c_{m+1}y\overrightarrow{c_{m+2}c_{m+1}}$ are extensions of C, respectively. If $c_mc_{m+2} \in E(G_2)$ we consider two cases to show that C is extendable.

Case 1: Suppose $|G_1| = 1$, that is, $V(G_1) = \{c_{m+1}\}$. Since G is $2-$connected, then $G \setminus c_{m+1} = G_2$ is connected. Let P be a path in G_2 from y to c_m. Since $V(C) \cap V(P) \neq \phi$

and $V(P) \not\subseteq V(C)$, then we can find a vertex $c_j \in V(C) \cap V(P)$ and $\hat{y} \in V(P) \setminus V(C)$ such that $\hat{y}c_j \in E(P)$. Thus, an extension of C is either the cycle $c_j\hat{y}c_{m+1}\overrightarrow{c_{j+1}c_m}c_{m+2}c_j$ if $j \neq m$ or the cycle $c_j\hat{y}\overrightarrow{c_{m+1}c_j}$ if $j = m$.

Case 2: Suppose $|G_1| > 1$. Then there exists some $0 \leq j \leq k-1$ such that $c_j \in V(G_1)$ and $c_j \neq c_{m+1}$. Also, either $j+1 \leq m$ or $m+2 \leq j-1$ since no edge of C is in G_1 (see Figure 2). Therefore, an extension of C is either the cycle $C' = c_jyc_{m+1}\overrightarrow{c_{j+1}c_m}\overrightarrow{c_{m+2}c_j}$ if $j+1 < m$ or the cycle $C'' = c_{m+1}y\overrightarrow{c_jc_m}\overrightarrow{c_{m+2}c_{j-1}}c_{m+1}$ if $m+2 > j-1$. In the case that $j+1 = m$, we replace $\overrightarrow{c_{j+1}c_m}$ in C' with vertex c_m. Similarly, in the case $m+2 = j-1$, we replace $\overrightarrow{c_{m+2}c_{j-1}}$ in C'' with vertex c_{m+2}.

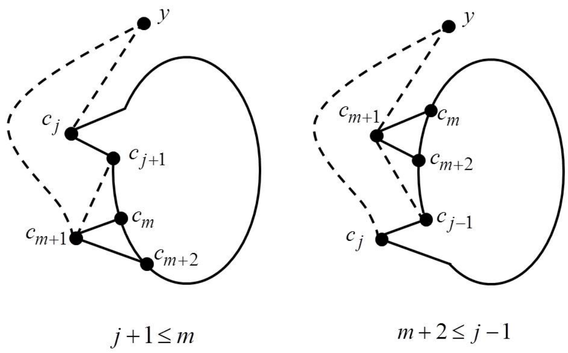

Figure 2: Case 2: $|G_1| > 1$

The initial remark in this section and the previous two claims complete the proof of Theorem 1.4. The proof of Corollary 1.1 is given below.

Proof: It is easy to see that if G_1 and G_2 are connected then every vertex of G is in some triangle since G is a join. Suppose G_1 is connected and G_2 is not connected. Since G is claw-free then G_2 can have at most two components. Furthermore, because G is a join then the disjoint components are K_m and K_n, i.e $G_2 = K_m \cup K_n$ for some m and n. Note that since G is 2−connected, then $|G_1| > 1$. Thus every vertex of G_1 is in P_2. Consequently, every vertex of G_1 and every vertex of $K_m \cup K_n$ is in some triangle in G since G is a join.

Now, suppose both G_1 and G_2 are not connected. Since G is $K_{1,3}$−free then $G = (K_m \cup K_n) + (K_r \cup K_s)$ for some m, n, r and s. A vertex in $K_m \cup K_n$ is in some triangle in G if and only if either the vertex is adjacent to another vertex in $K_m \cup K_n$ or at least one of r or s is greater than 1. Similarly, a vertex in $K_r \cup K_s$ is in some triangle in G if and only if either the vertex is adjacent to another vertex in $K_r \cup K_s$ or at least one of m or n is greater than 1. Thus, if $G \neq (K_1 \cup K_1) + (K_1 \cup K_s)$, then a vertex in G is in some triangle in G.

Conversely, if $G = (K_1 \cup K_1) + (K_1 \cup K_s)$ then the isolated vertex in $K_1 \cup K_s$, the second graph in the join, is not in any triangle of G. Hence G is not fully cycle extendable.

3 Proof of Theorem 1.5

Throughout this section, we let $G = G_1 + G_2$ be a 3-connected $K_{1,3}$−free graph and $P = p_0p_1...p_l$ be a non-Hamiltonian path of G. If $G = (u \cup K_n) + (v \cup K_s)$ for some integers

$n, s \geq 2$, then G is not path extendable since uv has no extension in G. Also, if $G = (u \cup K_n) + (vw \cup K_s)$ for some integers $n \geq 2$ and $s \geq 1$, then G is not path extendable since vuw has no extension in G. For the rest of this section, we assume G is not $(K_1 \cup K_n) + (K_r \cup K_s)$ for any positive integers n, r, s such that $n \geq 2$, $r \leq 2$ and $r + s \geq 3$.

First, we claim that P is extendable if its length is either 1 or 2. To see this, consider two cases. If an edge of P is in either G_1 or G_2 then P is clearly extendable because of the join structure of G. On the other hand, suppose P has no edge in either G_1 or G_2, that is, suppose WLOG, $p_0 \in V(G_1)$ and $p_1 \in V(G_2)$ (implying $p_2 \in V(G_1)$, in the case $l = 2$). If $l = 1$, note that there is an $x \in V(G_1)$ with $xp_0 \in E(G)$ or a $y \in V(G_2)$ such that $yp_1 \in E(G)$. Similarly, if $l = 2$, we either have $u \in V(G_1)$ such that either $up_0 \in E(G)$ or $up_2 \in E(G)$; or $v \in V(G_2)$ such that $vp_1 \in E(G)$. In either case, because $G = G_1 + G_2$, P is extendable.

If the length of P is 3 or more, we use similar cases considered in the previous section. If an edge of P is in G_1 and a vertex of G_2 is not in P, then it easy to verify that P is extendable because G is a join of G_1 and G_2. It is left to show that P is extendable if either no edge of P is in either G_1 or G_2, or no edge of P is in G_1 but all vertices of G_1 is in P for $l > 3$. The proof for these two cases are presented in Claim 3.1, Claim 3.2 and Claim 3.4.

Claim 3.1. *If no edge of P is in either G_1 or G_2, then P is extendable for $l \geq 3$.*

Proof: From the assumption, there exist a $t \leq l - 3$ such that $p_t, p_{t+2} \in V(G_i)$ and $p_{t+1}, p_{t+3} \in V(G_j)$, $i \neq j$. Moreover, there exists a vertex, say x, either in G_1 or G_2, that is not in P. WLOG, suppose $p_t \in V(G_1)$. If $x \in V(G_1)$, since G is $K_{1,3}$−free and $G = G_1 + G_2$ then at least one pair of vertices in $\{x, p_t, p_{t+2}\}$ are adjacent. If $xp_t \in E(G)$ or $xp_{t+2} \in E(G)$, then $p_0p_1...p_txp_{t+1}...p_l$ or $p_0p_1...p_{t+1}xp_{t+2}...p_l$, respectively are extensions of P. If $p_tp_{t+2} \in E(G)$ then $p_0p_1...p_tp_{t+2}p_{t+1}xp_{t+3}...p_l$ is an extension of P. It can be shown in a similar manner that P is extendable when $x \in V(G_2)$.

Claim 3.2. *If no edge of P is in G_1 but all vertices of G_1 is in P and $xp_t \notin E(G_2)$ for any $x \in V(G_2) \setminus V(P)$ and any $t \leq l$, then P is extendable.*

Proof: Since $xp_t \notin E(G_2)$ for any $x \in V(G_2) \setminus V(P)$ then G_2 is not connected, i.e. $G_2 = K_m \cup K_n$ where $G[V(G_2) \setminus V(P)] = K_m$ and $G[V(G_2) \cap V(P)] = K_n$. It will also follow that $|G_1| \geq 3$ since G is 3-connected and G_2 is not connected. Thus, because all vertices of G_1 is in P then there exist integers t, s and r with $0 \leq t < s - 1 < r - 1 \leq l - 1$ such that $p_t, p_s, p_r \in V(G_1)$. Since no edge of P is in G_1 then $p_{t+1}, p_{s-1}, p_{s+1}, p_{r-1} \in V(G_2)$. Note that $p_{s-1}p_{s+1} \in E(G)$ since $G[V(G_2) \cap V(P)] = K_n$. Thus, the path $p_0p_1...p_txp_sp_{t+1}p_{t+2}...p_{s-1}p_{s+1}...p_r...p_l$ is an extension of P.

Finally, to complete the proof of Theorem 2.1, we show that P is extendable when there is an $x \in V(G_2) \setminus V(P)$ such that $xp_t \in E(G_2)$ for some $t \leq l$. To prove this, we need the following claim.

Claim 3.3. *Let $Q = q_0q_1...q_r$ be a non-Hamiltonian path of G_1 with $r \geq 2$. Suppose there exists a $y \in V(G_1) \setminus V(Q)$ such that $yq_i \in E(G_1)$ for some $0 < i < r$. Then either Q has an extension in G_1 containing y or $q_sq_t \in E(G)$ for any $0 \leq s < i < t \leq r$.*

Proof: Let $y \in V(G_1) \setminus V(Q)$ such that $yq_i \in E(G_1)$ for some $0 < i < r$. Also, suppose $q_sq_t \notin E(G)$ for some s and t such that $0 \leq s < i < t \leq r$. It is not difficult to see that Q will have an extension containing y when $r = 2$. For $r \geq 3$ we consider two cases. The first case is when $q_{i-1}q_{i+1} \notin E(G)$. Then either yq_{i-1} or yq_{i+1} is in $E(G)$ because G is $K_{1,3}$−free.. Thus, $q_0q_1q_syq_i...q_r$ or $q_0q_1...q_iyq_t..q_r$ are extensions of Q.

Now, suppose $q_{i-1}q_{i+1} \in E(G)$. Then either $s < i-1$ or $t > i+1$. For $k = i+1, i+2, ..., t$, let m_k be the largest integer such that $q_{i-m_k}q_j \in E(G_1)$ for all $i+1 \leq j \leq k$. Similarly, for $l = i-1, i-2, ..., s$, define n_l to be the largest integer such that $q_jq_{i+n_l} \in E(G_1)$ for all $l \leq j \leq i-1$. Note that both sequences $\{m_k\}_{k=i+1}^{k=t}$ and $\{n_l\}_{l=i-1}^{l=s}$ are non-increasing and bounded below by 1. Now, choose m and n as the first indices of $\{m_k\}_{k=i+1}^{k=t}$ and $\{n_l\}_{l=i-1}^{l=s}$ where each achieves its minimum value, respectively. Hence, we have $q_{i-m-1}q_{i+n+1} \notin E(G_1)$ and $q_uq_v \in E(G_1)$ for $i-m \leq u \leq i-1$ and $i+1 \leq v \leq i+n$.

Now, because G is $K_{1,3}-$free, then either $yq_{i-m-1} \in E(G_1)$ or $yq_{i+n+1} \in E(G_1)$. If $yq_{i-m-1} \in E(G_1)$ then $q_0q_1...q_{i-m-1}yq_iq_{i-1}...q_{i-m}q_{i+1}q_{i+2}...q_r$ is an extension of Q in G_1. Similarly, if $yq_{i+n+1} \in E(G_1)$ then an extension of Q in G_1 is given by

$$q_0q_1...q_{i-1}q_{i+n}q_{i+n-1}...q_iyq_{i+n+1}q_{i+n+2}...q_r.$$

Claim 3.4. *If no edge of P is in G_1 but all vertices of G_1 is in P and there exist an $x \in V(G_2) \setminus V(P)$ such that $xp_t \in E(G_2)$ for some $t \leq l$, then P is extendable.*

Proof: It is easy to see that if $0 \leq t \leq l$ and either p_{t-1} or p_{t+1} is in G_1 then P is extendable because of G being a join of two graphs.

Suppose $0 < t < l$ and $p_{t-1}, p_{t+1} \notin V(G_1)$. Let Q be a longest subpath of P in G_2 containing p_t. Note that $|Q| \geq 3$. Suppose Q is a path from p_r to p_s. From the previous claim, Q has an extension in G_2 which contains x or $p_{r'}p_{s'} \in E(G)$ for any r', s' satisfying $r \leq r' < t < s' \leq s$. Moreover, by the maximality of the length of Q, $p_{r-1}, p_{s+1} \in V(G_1)$, or equivalently, $xp_{r-1}, xp_{s+1} \in E(G)$. If Q has an extension in G_2 then clearly, P is extendable. If $p_{r'}p_{s'} \in E(G)$ for any r', s' such that $r \leq r' < t < s' \leq s$ then the following are extensions of P:

$$\begin{cases} p_0p_1...p_{r-1}xp_tp_{t-1}...p_rp_{t+1}...p_l & \text{if } r \neq 0 \\ p_0p_sp_{s-1}...p_{t+1}p_1p_2...p_txp_{s+1}p_{s+2}...p_l & \text{if } r = 0, s \neq l \\ p_0p_1...p_{t-1}p_sp_{s-1}...p_txp_{s+1}...p_l & \text{if } s \neq l \\ p_0p_1...p_{r-1}xp_tp_{t+1}...p_{l-1}p_{t-1}p_{t-2}...p_rp_l & \text{if } s = l, r \neq 0 \end{cases}$$

Note that it cannot be the cases that $r = 0$ and $s = l$. Hence, in any case, P is extendable when $0 < t < l$ and $p_{t-1}, p_{t+1} \notin V(G_1)$.

Finally it remains to show that P is extendable when $t = 0$ and $p_1 \in V(G_2)$ and when $t = l$ and $p_{l-1} \in V(G_2)$. Suppose $t = 0$ and $p_1 \in V(G_2)$. If $|G_1| = 1$ then G_2 is 2-connected and $G_2 \setminus p_0$ is connected. We claim that there exists a $y \in V(G_2) \setminus V(P)$ such that $yp_i \in E(G_2)$ for some $0 < i < l$. To see this, suppose otherwise. Then for every $y \in V(G_2) \setminus V(P)$ there is a path in G_2 from y to p_i for every $0 < i < l$ not containing yp_i as an edge since $G_2 \setminus p_0$ is connected. This implies that each path contains xp_l for some $x \in V(G_2) \setminus V(P)$. In particular, $xp_lp_{l-1}...p_1$ is in G_2. Thus P is contained in G_2, which is a contradiction.

Since there exists a $y \in V(G_2) \setminus V(P)$ such that $yp_i \in E(G_2)$ for some $0 < i < l$, using the previous claim and the argument above, P is extendable. If $|G_1| \geq 2$ then there exists an r'' with $r'' > 1$ and $p_{r''-1}, p_{r''+1} \in V(G_2)$. Since G is $K_{1,3}-$free then at least one pair of vertices in $\{x, p_{r''-1}, p_{r''+1}\}$ are adjacent. If $p_{r''-1}p_{r''+1} \in E(G)$ then $p_0xp_{r''}p_1p_2...p_{r''-1}p_{r''+1}...p_l$ is an extension of P. If $xp_{r''-1} \in E(G)$ or $xp_{r''+1}$ then $p_0p_1...p_{r''-1}xp_{r''}p_{r''+1}..p_l$ or $p_0p_1...p_{r''}xp_{r''+1}...p_l$, respectively, are extensions of P. It can be shown in a similar manner that P is extendable when $t = l$ and $p_{l-1} \in V(G_2)$.

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